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Helmholtz, Hermann von. 1860-64. On the Motion of the Strings of a Violin.
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December 19, 1860.—*The President in the Chair.*

Mr. Alexander Duncan, Mr. James Anderson Snell, and Mr. Robert H. Penman, were elected members.

On the Motion of the Strings of a Violin. By PROFESSOR H. HELMHOLZ, Heidelberg, Honorary Member of the Society. Communicated by PROFESSOR WILLIAM THOMSON.

I HAVE been studying for some time the causes of the different qualities of sound, and as I found that those differences depended principally upon the number and intensity of the harmonic sounds, accompanying the fundamental one, I was obliged to investigate the forms of elastic vibrations performed by different sounding bodies. Among such vibrations, the form of which is not yet exactly known, the vibrations of strings excited by the bow of a violin are peculiarly interesting. Th. Young describes them as very irregular; but I suppose that his assertion relates only to the motions which remain after the impulse of the bow has ceased. At least, I myself found the motion very regular as long as the bow is applied near one end of the string, in the regular way commonly followed by players of the violin. I used a method of observing very similar to that of Lissajous. Already, without the assistance of any instruments, one can see easily that a string moved by the bow vibrates in one plane only—the same plane in which the string itself and the hairs of the bow are situated. This plane was horizontal in my experiments. The string was powdered with starch, and strongly illuminated. One of the little grains of starch, looking like a bright point, was observed by a vertical microscope, the object lens of which was fixed to one of the branches of a tuning fork. The fork, making 120 vibrations in the second, was placed between the branches of a horse-shoe electro-magnet, which was magnetized by an interrupted electric current, the number of interruptions being itself 120 in the second. In that way the fork was kept vibrating for as long a time as I desired. The lens of the microscope vibrated in a direction parallel to the string, and therefore perpendicular to its vibrations. The string I used was the third string of a violin, answering to the note A, tuned a little higher than common, to 480 vibrations, and, therefore, it performed four vibrations for every one of the tuning fork. Looking through the microscope, I observed the grain of starch describing an illuminated curved line, the horizontal abscissæ of which corresponded to the deviations of the tuning fork, and the vertical ordinates to the deviations of the string. I found it a matter of importance to use a violin of most perfect construction, and I had occasion to get a very fine instrument of Guadanini for these experiments. On the common instruments of inferior quality I could not keep the curve constant enough for numbering the little indentures, which I shall describe afterwards, although the general character of the curve was the same on all the instruments I tried. The curve used to move by jerks along the line of abscissæ, and every jerk was accompanied by a scratching noise of the bow. On the contrary, with the Italian instrument, and after some practice, I got a curve completely quiescent as long as the bow moved in one direction, the sound being very pure and free from scratching.

We may consider the motion of the string as being compounded of two different sets of vibrations, the first of which is the principal motion as to magnitude. Its period is equal to the period of the fundamental sound of the string, and it is independent of the situation of the point where the bow is applied. The second motion produces only very small indentures of the curve. Its period of vibration answers to one of the higher harmonics of the string. It is known that a string, when producing only one of its higher harmonics, is divided into several vibrating divisions, of equal length, being separated by quiescent points, which are called nodes. In all the nodes of the second motion of the string in the compound result at present considered, the principal motion appears alone; and also in the other points of the string the indentures corresponding to the second motion are easily obliterated, if the line of light is too broad.

The principal motion of the string is such that every point of it goes to one side with a constant velocity, and returns to the other side with another constant velocity.

Fig. 1 represents four such vibrations, corresponding to one vibration of the fork. The horizontal abscissæ are proportional to the time, the vertical ordinates to the deviation of the vibrating point. Every vibration is formed on the curve by two straight lines. The curve is not seen quite in the same way through the microscope, because there the horizontal abscissæ are not proportional to the time but to the sine of the time. It must be imagined that the curve, Fig. 1, is wound up

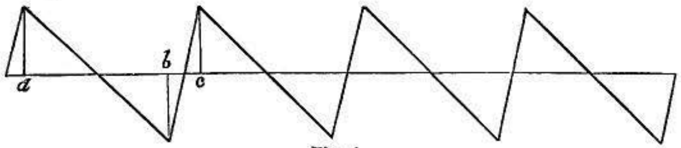


Fig. 1.

round a cylinder, so that the two ends of it meet together, and that the whole is seen in perspective from a great distance; thus is had the real appearance of the curve, as represented in two different positions in Fig. 2. If the number of vibrations of the string is accurately equal to four times the number of the tuning fork, the curve appears quietly keeping the same position. If there is, on the contrary, a little

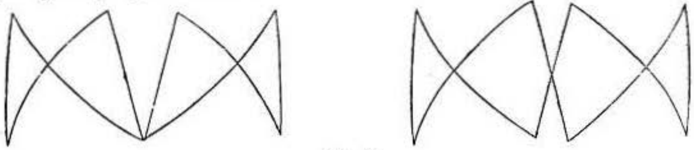


Fig. 2.

difference of tuning, it looks as if the cylinder rotates slowly about its axis, and by the motion of the curve the observer gets as lively an impression of a cylindrical surface, on which it seems to be drawn, as if looking to a stereoscopic picture. The same impression may be produced by combining, stereoscopically, the two diagrams of Fig. 2.

We learn, therefore, by the experiments,—

1. That the strings of a violin, when struck by the bow, vibrate in one plane.

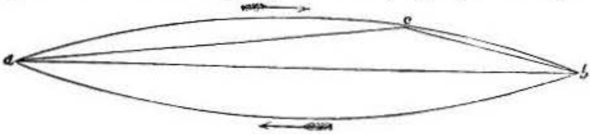
2. That every point of the string moves to and fro with two constant velocities.

These two data are sufficient for finding the complete equation of the motion of the whole string. It is the following:—

$$y = A \Sigma \left\{ \frac{1}{n^2} \sin \left(\frac{\pi n x}{l} \right) \sin \left(\frac{2 \pi n t}{T} \right) \right\} \text{............} 1.$$

y is the deviation of the point, whose distance from one end of the string is x; l, the length of the string; t, the time; T, the duration of one vibration; A, an arbitrary constant; and n, any whole number; and all values of the expression under the sign Σ, got in that way, are to be summed.

A comprehensive idea of the motion represented by this equation may be given in the following way:—Let *a b*, Fig. 3, be the equilibrium



Fig, 3.

position of the string. During the vibration its forms will be similar to *a c b*, compounded of two straight lines, *a c* and *c b*, intersecting in the point *c*. Let this point of intersection move with a constant velocity along two flat circular arcs, lying symmetrically on the two sides of the string, and passing through its ends, as represented in Fig. 3. A motion the same as the actual motion of the whole string is thus given.

As for the motion of every single point, it may be deduced from equation (1.), that the two parts *a b* and *b c*, Fig. 1, of the time of every vibration are proportional to the two parts of the string which are separated by the observed point. The two velocities of course are inversely proportional to the times $a\,b$ and $b\,c$. In that half of the string which is touched by the bow, the smaller velocity has the same direction as the bow; in the other half of the string it has the contrary direction. By comparing the velocity of the bow with the velocity of the point touched by it, I found that this point of the string adheres fast to the bow and partakes in its motion during the time $a\,b$, then is torn off and jumps back to its first position during the time $b\,c$, till the bow again gets hold of it.

With these principal vibrations smaller vibrations are compounded, the nature of which I can define accurately only in the case where the bow touches a point whose distance from the nearer end of the string is $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, &c., of its whole length, or generally $\frac{1}{m}$, if m is a whole number. Because the point where the bow is applied is not moved by any vibration belonging to the mth, $2m$th, &c., harmonic, it is quite indifferent for the motion of that point, and for the impulses exerted by the bow upon the string, whether vibrations corresponding to the mth harmonic, exist or not. Th. Young has proved that if we excite the vibrations of a string by bending it with the finger, as in the harp, or hit it with a single stroke, as in the piano, in the ensuing motion all those harmonics are wanting which have a node in the touched point. I concluded, therefore, that also the bow cannot excite those harmonics which have a node at the point where it is applied, and I found indeed, if this point is distant $\frac{1}{m}$ from the end, that the ear does not hear the mth harmonic sound, although it distinguishes very well all the other harmonics. Therefore, in the equation (1.), all those members of the sum will be wanting in which n is equal to m, or $2m$, or $3m$, &c. These members, taken together, constitute a vibration of the string with m vibrating divisions. Every such division performs the same form of vibration we have described as the principal vibration of the whole string. These small vibrations must be subtracted from the principal vibration of the whole string for getting its actual vibration. Curves constructed according to this theoretical view represent very well the really observed curves. If $m = 6$ and the observed point is distant $\frac{1}{12}$ from the other end of the string, the motion is represented in Fig. 4. Near the end of the string, where the bow is commonly applied by players, the nodes of different harmonics are very near to each other, so that the bow is nearly always at, or at least very near to, the place of a node. Striking in the middle between two nodes, I could not get a curve sufficiently constant for my observations. If I strike very near the end, the sound changes often between the fundamental and the

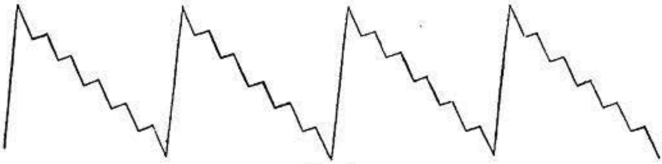


Fig. 4.

second or third harmonic, which is indicated by gradual corresponding alterations of the curve.

January 16, 1861.—*The President in the Chair.*

Mr. John Addie and Mr. John W. Stone were elected members.

Some results in Electro-Magnetism, obtained with the Balance Galvanometer. By George Blair, M.A.

Since bringing the Balance Galvanometer, along with some other apparatus, before the Society in the course of last session, the writer had made some experiments with this new galvanometer, which led to

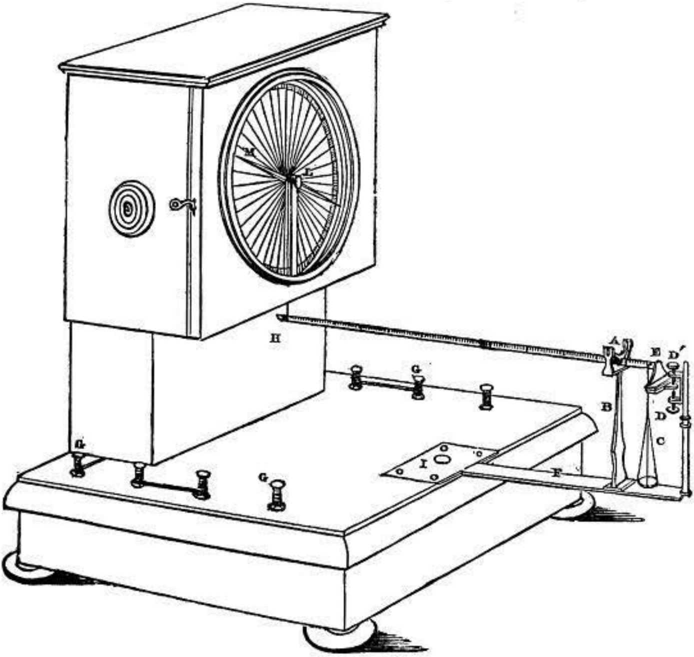


Fig. 1.